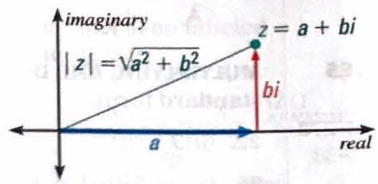
KEY CONCEPT

For Your Notebook

Absolute Value of a Complex Number

The **absolute value** of a complex number $z = a + bi$, denoted $|z|$, is a nonnegative real number defined as $|z| = \sqrt{a^2 + b^2}$. This is the distance between z and the origin in the complex plane.



EXAMPLE 7 Find absolute values of complex numbers

Find the absolute value of (a) $-4 + 3i$ and (b) $-3i$.

a. $|-4 + 3i| = \sqrt{(-4)^2 + 3^2} = \sqrt{25} = 5$

b. $|-3i| = |0 + (-3i)| = \sqrt{0^2 + (-3)^2} = \sqrt{9} = 3$

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✓ GUIDED PRACTICE for Examples 6 and 7

Plot the complex numbers in the same complex plane. Then find the absolute value of each complex number.

15. $4 - i$ **16.** $-3 - 4i$ **17.** $2 + 5i$ **18.** $-4i$

4.6 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS8 for Exs. 11, 29, and 67

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 21, 50, 60, 69, and 74

SKILL PRACTICE

1. VOCABULARY What is the complex conjugate of $a - bi$?

2. ★ WRITING Is every complex number an imaginary number? *Explain.*

EXAMPLE 1 on p. 275 for Exs. 3–11

SOLVING QUADRATIC EQUATIONS Solve the equation.

3. $x^2 = -28$ **4.** $r^2 = -624$ **5.** $z^2 + 8 = 4$

6. $s^2 - 22 = -112$ **7.** $2x^2 + 31 = 9$ **8.** $9 - 4y^2 = 57$

9. $6t^2 + 5 = 2t^2 + 1$ **10.** $3p^2 + 7 = -9p^2 + 4$ **11.** $-5(n - 3)^2 = 10$

EXAMPLE 2 on p. 276 for Exs. 12–21

ADDING AND SUBTRACTING Write the expression as a complex number in standard form.

12. $(6 - 3i) + (5 + 4i)$ **13.** $(9 + 8i) + (8 - 9i)$ **14.** $(-2 - 6i) - (4 - 6i)$

15. $(-1 + i) - (7 - 5i)$ **16.** $(8 + 20i) - (-8 + 12i)$ **17.** $(8 - 5i) - (-11 + 4i)$

18. $(10 - 2i) + (-11 - 7i)$ **19.** $(14 + 3i) + (7 + 6i)$ **20.** $(-1 + 4i) + (-9 - 2i)$

21. ★ **MULTIPLE CHOICE** What is the standard form of the expression $(2 + 3i) - (7 + 4i)$?

Ⓐ -4 Ⓑ $-5 + 7i$ Ⓒ $-5 - i$ Ⓓ $5 + i$

EXAMPLES 4 and 5 on pp. 277–278 for Exs. 22–33

MULTIPLYING AND DIVIDING **Write the expression as a complex number in standard form.**

22. $6i(3 + 2i)$
23. $-i(4 - 8i)$
24. $(5 - 7i)(-4 - 3i)$
25. $(-2 + 5i)(-1 + 4i)$
26. $(-1 - 5i)(-1 + 5i)$
27. $(8 - 3i)(8 + 3i)$
28. $\frac{7i}{8 + i}$
29. $\frac{6i}{3 - i}$
30. $\frac{-2 - 5i}{3i}$
31. $\frac{4 + 9i}{12i}$
32. $\frac{7 + 4i}{2 - 3i}$
33. $\frac{-1 - 6i}{5 + 9i}$

EXAMPLE 6 on p. 278 for Exs. 34–41

PLOTTING COMPLEX NUMBERS **Plot the numbers in the same complex plane.**

34. $1 + 2i$
35. $-5 + 3i$
36. $-6i$
37. $4i$
38. $-7 - i$
39. $5 - 5i$
40. 7
41. -2

EXAMPLE 7 on p. 279 for Exs. 42–50

FINDING ABSOLUTE VALUE **Find the absolute value of the complex number.**

42. $4 + 3i$
43. $-3 + 10i$
44. $10 - 7i$
45. $-1 - 6i$
46. $-8i$
47. $4i$
48. $-4 + i$
49. $7 + 7i$

50. ★ **MULTIPLE CHOICE** What is the absolute value of $9 + 12i$?

Ⓐ 7   Ⓑ 15   Ⓒ 108   Ⓓ 225

STANDARD FORM **Write the expression as a complex number in standard form.**

51. $-8 - (3 + 2i) - (9 - 4i)$
52. $(3 + 2i) + (5 - i) + 6i$
53. $5i(3 + 2i)(8 + 3i)$
54. $(1 - 9i)(1 - 4i)(4 - 3i)$
55. $\frac{(5 - 2i) + (5 + 3i)}{(1 + i) - (2 - 4i)}$
56. $\frac{(10 + 4i) - (3 - 2i)}{(6 - 7i)(1 - 2i)}$

ERROR ANALYSIS ***Describe*** **and correct the error in simplifying the expression.**

57.
$$(1 + 2i)(4 - i)$$
$$= 4 - i + 8i - 2i^2$$
$$= -2i^2 + 7i + 4$$

58.
$$|2 - 3i| = \sqrt{2^2 - 3^2}$$
$$= \sqrt{-5}$$
$$= i\sqrt{5}$$

59. **ADDITIVE AND MULTIPLICATIVE INVERSES** The additive inverse of a complex number z is a complex number z_a such that $z + z_a = 0$. The multiplicative inverse of z is a complex number z_m such that $z \cdot z_m = 1$. Find the additive and multiplicative inverses of each complex number.

a. $z = 2 + i$ b. $z = 5 - i$ c. $z = -1 + 3i$

60. ★ **OPEN-ENDED MATH** Find two imaginary numbers whose sum is a real number. How are the imaginary numbers related?

CHALLENGE **Write the expression as a complex number in standard form.**

61. $\frac{a + bi}{c + di}$
62. $\frac{a - bi}{c - di}$
63. $\frac{a + bi}{c - di}$
64. $\frac{a - bi}{c + di}$

○ = **WORKED-OUT SOLUTIONS** on p. WS1

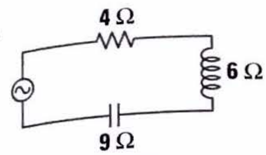
★ = **STANDARDIZED TEST PRACTICE**

PROBLEM SOLVING

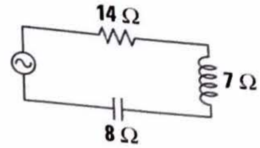
EXAMPLE 3 on p. 277 for Exs. 65–67

CIRCUITS In Exercises 65–67, each component of the circuit has been labeled with its resistance or reactance. Find the impedance of the circuit.

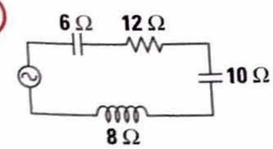
65.



66.



67.



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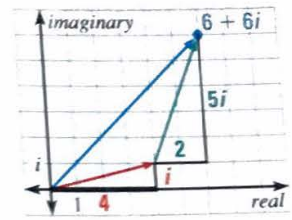
68. VISUAL THINKING The graph shows how you can geometrically add two complex numbers (in this case, $4 + i$ and $2 + 5i$) to find their sum (in this case, $6 + 6i$). Find each of the following sums by drawing a graph.

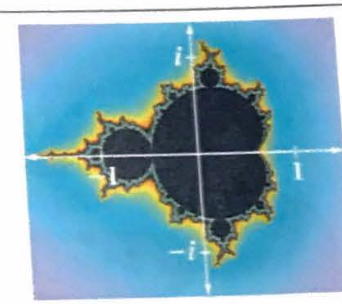


a. $(5 + i) + (1 + 4i)$
b. $(-7 + 3i) + (2 - 2i)$
c. $(3 - 2i) + (-1 - i)$
d. $(4 + 2i) + (-5 - 3i)$

69. ★ SHORT RESPONSE Make a table that shows the powers of i from i^1 to i^8 in the first row and the simplified forms of these powers in the second row. *Describe* the pattern you observe in the table. Verify that the pattern continues by evaluating the next four powers of i.

In Exercises 70–73, use the example below to determine whether the complex number c belongs to the Mandelbrot set. *Justify* your answer.

EXAMPLE Investigate the Mandelbrot set



The Mandelbrot set is the black region in the complex plane above.

Consider the function $f(z) = z^2 + c$ and this infinite list of complex numbers: $z_0 = 0$, $z_1 = f(z_0)$, $z_2 = f(z_1)$, $z_3 = f(z_2)$, ... If the absolute values of $z_0, z_1, z_2, z_3, \ldots$ are all less than some fixed number N, then c belongs to the *Mandelbrot set*. If the absolute values become infinitely large, then c does not belong to the Mandelbrot set.

Tell whether $c = 1 + i$ belongs to the Mandelbrot set.

Solution

Let $f(z) = z^2 + (1 + i)$.

$z_0 = 0$	$\lvert z_0\rvert = 0$
$z_1 = f(0) = 0^2 + (1 + i) = 1 + i$	$\lvert z_1\rvert \approx 1.41$
$z_2 = f(1 + i) = (1 + i)^2 + (1 + i) = 1 + 3i$	$\lvert z_2\rvert \approx 3.16$
$z_3 = f(1 + 3i) = (1 + 3i)^2 + (1 + i) = -7 + 7i$	$\lvert z_3\rvert \approx 9.90$
$z_4 = f(-7 + 7i) = (-7 + 7i)^2 + (1 + i) = 1 - 97i$	$\lvert z_4\rvert \approx 97.0$

▶ Because the absolute values are becoming infinitely large, $c = 1 + i$ does not belong to the Mandelbrot set.

70. $c = i$
71. $c = -1 + i$
72. $c = -1$
73. $c = -0.5i$

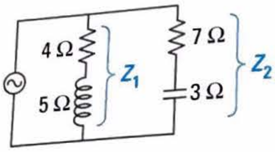
74. ★ **SHORT RESPONSE** Evaluate $\sqrt{-4} \cdot \sqrt{-25}$ and $\sqrt{100}$. Does the rule $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$ on page 266 hold when a and b are negative numbers?

75. **PARALLEL CIRCUITS** In a *parallel circuit*, there is more than one pathway through which current can flow. To find the impedance Z of a parallel circuit with two pathways, first calculate the impedances Z_1 and Z_2 of the pathways separately by treating each pathway as a series circuit. Then apply this formula:

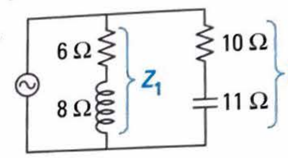
$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

What is the impedance of each parallel circuit shown below?

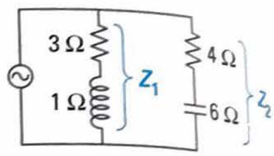
a.



b.



c.



76. **CHALLENGE** *Julia sets*, like the Mandelbrot set shown on page 281, are fractals defined on the complex plane. For every complex number c, there is an associated Julia set determined by the function $f(z) = z^2 + c$.

For example, the Julia set corresponding to $c = 1 + i$ is determined by the function $f(z) = z^2 + 1 + i$. A number z_0 is a member of this Julia set if the absolute values of the numbers $z_1 = f(z_0)$, $z_2 = f(z_1)$, $z_3 = f(z_2)$, . . . are all less than some fixed number N, and z_0 is not a member if these absolute values grow infinitely large.

A Julia set

Tell whether the given number z_0 belongs to the Julia set associated with the function $f(z) = z^2 + 1 + i$.

a. $z_0 = i$ **b.** $z_0 = 1$ **c.** $z_0 = 2i$ **d.** $z_0 = 2 + 3i$

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77. There are 185 students in this year's freshman class. What additional information is needed to predict the number of students in next year's freshman class?

Ⓐ The rate of change in the number of students in the freshman class

Ⓑ The number of females in this year's freshman class

Ⓒ The number of students in this year's senior class

Ⓓ The maximum number of students in the school

78. What are the slope m and y-intercept b of the line that contains the point $(-4, 1)$ and has the same y-intercept as $3x - 2y = 10$?

Ⓐ $m = -\frac{3}{2}, b = -5$ Ⓑ $m = 1, b = 5$

Ⓒ $m = \frac{3}{2}, b = 7$ Ⓓ $m = \frac{9}{4}, b = 10$

EXTRA PRACTICE for Lesson 4.6, p. 1013 **ONLINE QUIZ** at classzone.com